Tysoe Calculations Policy

This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.
Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum.

Concrete representation – children are first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.

Pictorial representation – children have sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation – children are now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations as new concepts are introduced. This is an approach for all children, not a way of differentiating for lower attaining children.

Real things and structured images enable children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Debbie Morgan (2016)

At Tysoe, we use a variety of images and representations to support the teaching of different concepts.

Part-Whole Model

The part-whole model supports children in their understanding of aggregation and partitioning.

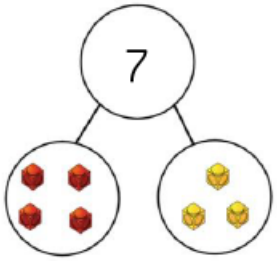
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

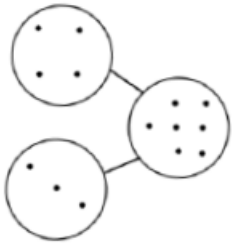
Concrete



$7 = 4 + 3$

$7 = 3 + 4$

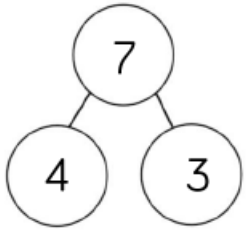
Pictorial

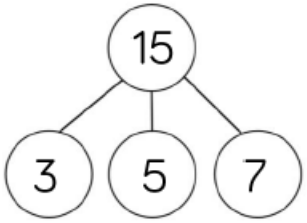


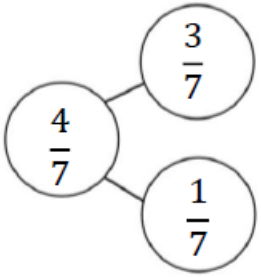
$7 - 3 = 4$

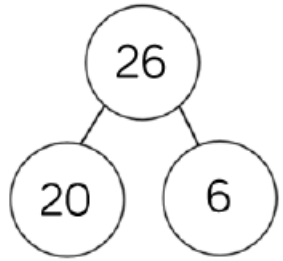
$7 - 4 = 3$

Abstract







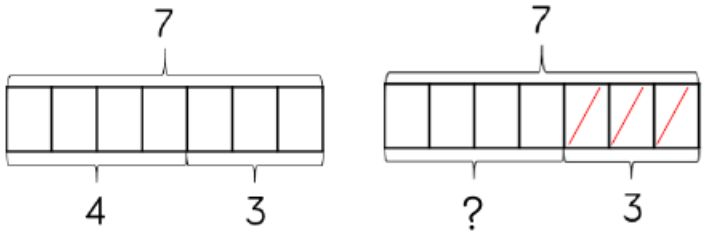


Bar Model

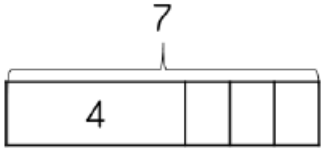
The bar model is another type of part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in line as a concrete representation of the bar model.

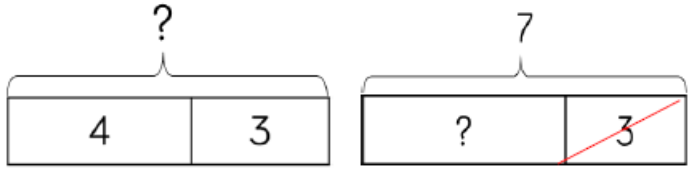
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.



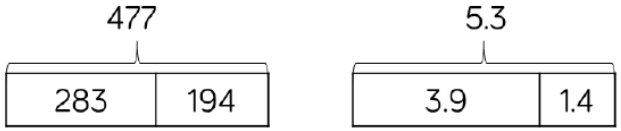
A combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.



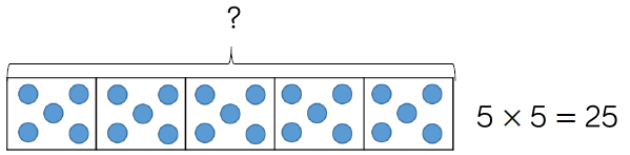
Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.



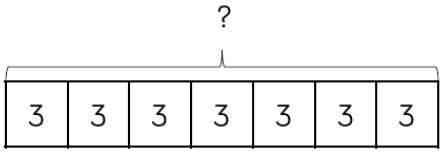
In KS2, children can use bar models to represent larger numbers, decimals and fractions.



Children can use the bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.



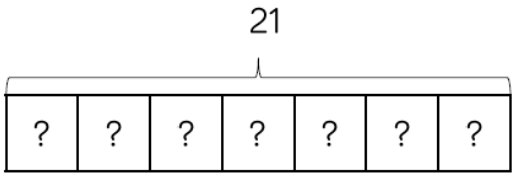
$5 \times 5 = 25$



$3 \times 7 = 21$

$7 \times 3 = 21$

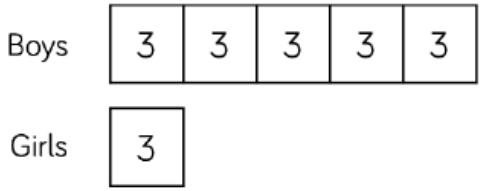
Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.



$21 \div 7 = 3$

It is important when solving word problems that the bar model represents the problem.

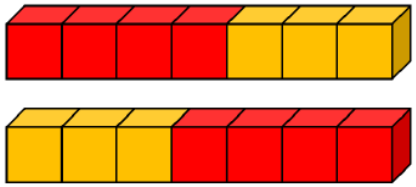
KS2 children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there? Using more than one bar model provides an opportunity to compare the groups.



Cubes

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

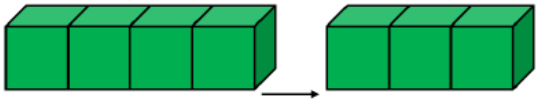
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.



$7 = 4 + 3$

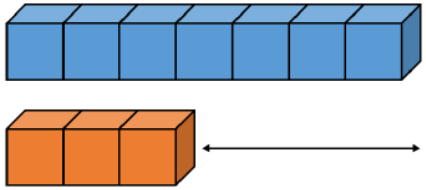
$7 = 3 + 4$

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take-away.



$7 - 3 = 4$

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between numbers.



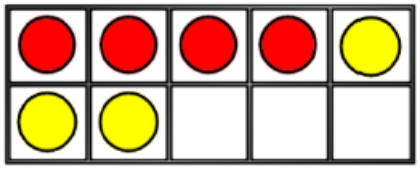
$7 - 3 = 4$

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

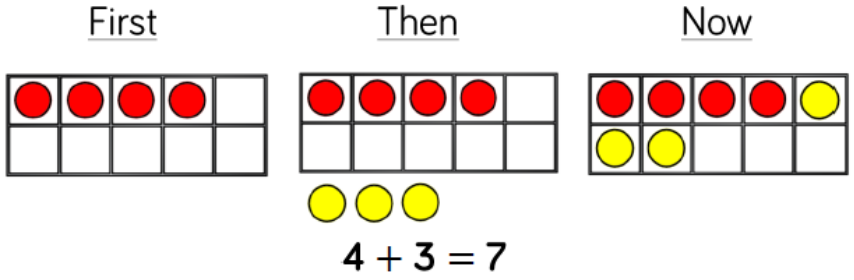
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

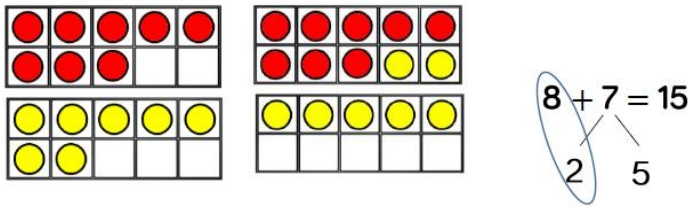
4 is a part.
3 is a part.
7 is the whole.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

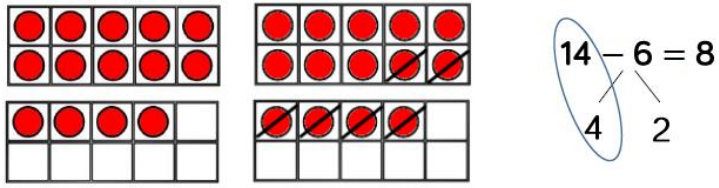


Ten Frames (within 20)

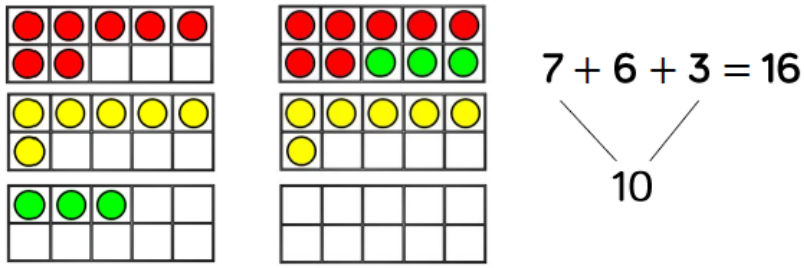
When adding two single digits, children can make each number on a separate ten frame before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.



When subtracting a one-digit number from a two digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10. This supports mental methods of subtraction.



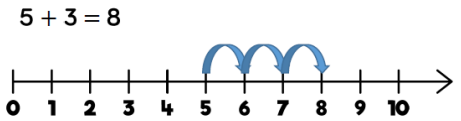
When adding three single-digits numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to make 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.



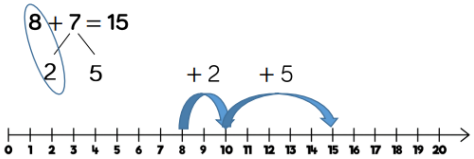
Number Lines (labelled)

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

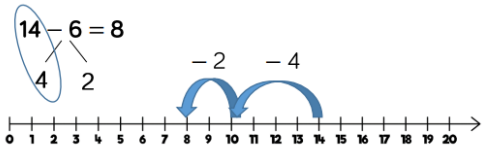
Children can start by counting on or back in ones, up or down the number line.



Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and then add on the remaining part.

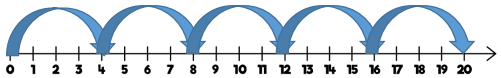


Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by 10 frames so children can see how they partition the smaller number into two separate jumps.

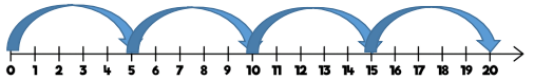


Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single digit multiplications. When multiplying the children start at 0 and then count on to find the product of the numbers.

4 X 5 = 20

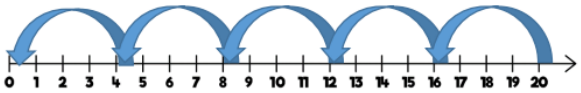


5 X 4 = 20



When dividing, start at the number they are dividing and then count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

20 ÷ 4 = 5



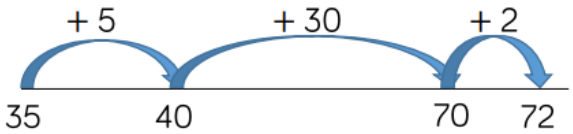
Labelled number lines can be useful with smaller multiples. However, they become inefficient as numbers become larger.

Number Lines (Blank)

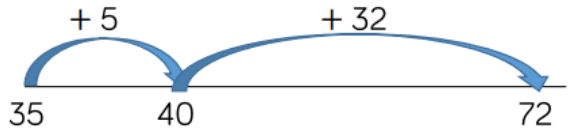
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

35 + 37 = 72

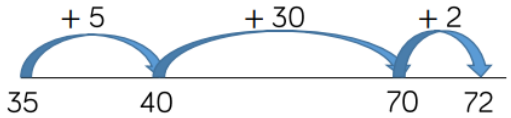


or



Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

72 – 35 = 37



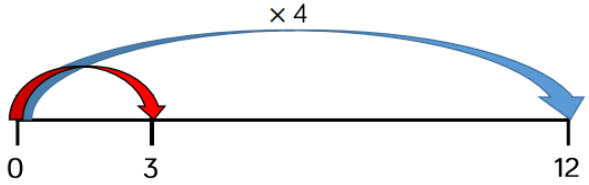
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and the counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Children can use blank number lines to represent scaling as multiplication or division. Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

A red car travels 3 miles.
A blue car travels 4 times further.
How far does the blue car travel?

Blank number lines without intervals can also be used for children to represent scaling.

A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

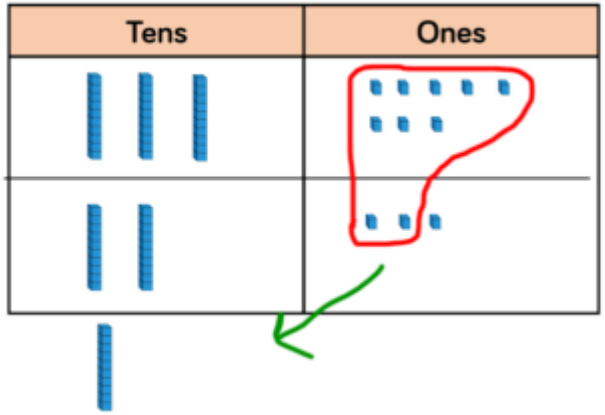


Base 10/Dienes (addition)

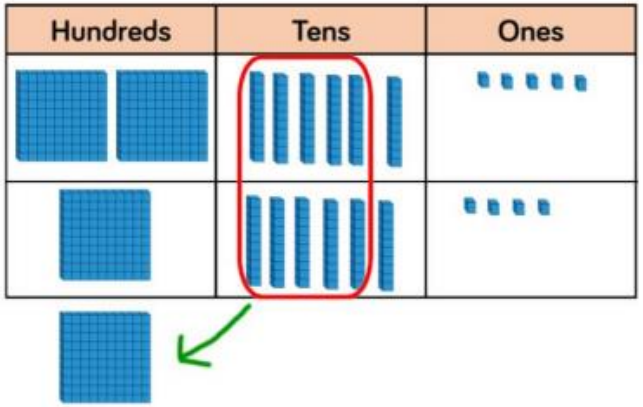
Using Base 10 or Dienes is an effective way to support children's understanding of column addition and subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so that they can see the clear links between the written method and the model.

Addition

Children should first add without regrouping before moving on to addition with regrouping. When adding, always start with the smallest place value. The representation becomes less efficient with larger numbers due to the size of Base 10.



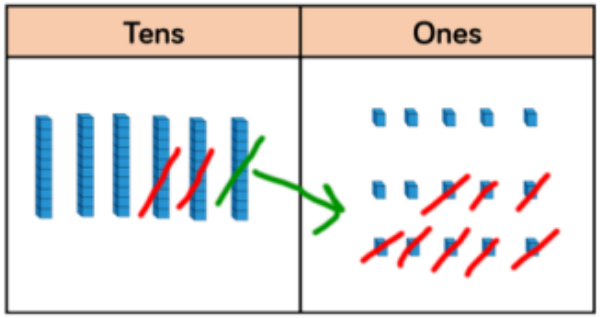
$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



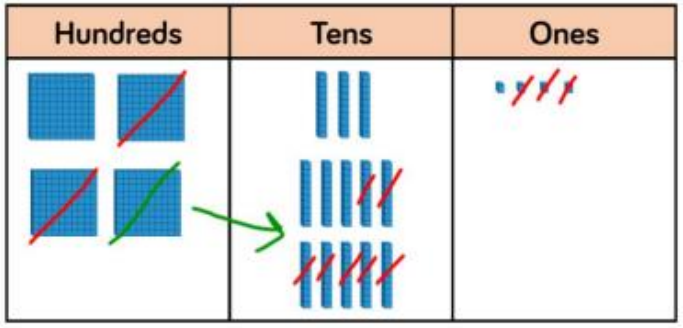
$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Subtraction

Children should first subtract without regrouping before moving on to subtraction with regrouping. When building the model, children should just make the minuend (top number) using base 10. They then subtract the other number. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently. The representation becomes less efficient with larger numbers due to the size of Base 10.



$$\begin{array}{r} {}^{5}\cancel{6}\,{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$

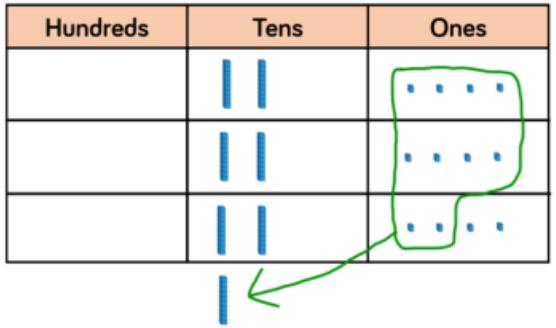


$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}35 \\ -\ 273 \\ \hline 262 \\ \hline \end{array}$$

Multiplication

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculations alongside the equipment so that they can see how the concrete and written representations match.

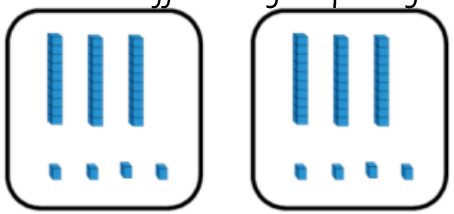
As the numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of regroups needed.



$$\begin{array}{r} {}^{1}24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$

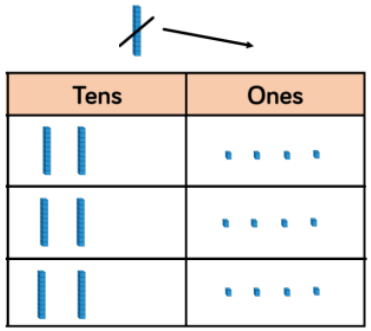
Division

Using Base 10 or Dienes is an effective way to support children's understanding of division. When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can share the Base 10/Dienes between different groups e.g. by drawing circles or rows on a place value grid.

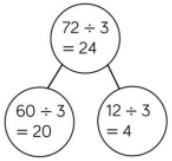


$$68 \div 2 = 34$$

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they regroup e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.



$$72 \div 3 = 24$$

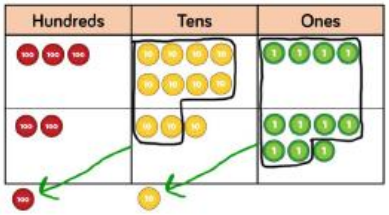


Place Value Counters

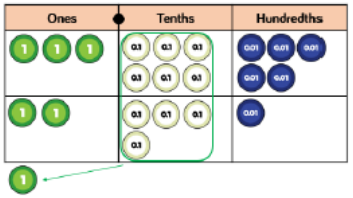
Using place value counters is an effective way to support children's understanding of column calculations. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Addition

Children should first add without regrouping before moving on to addition with regrouping.



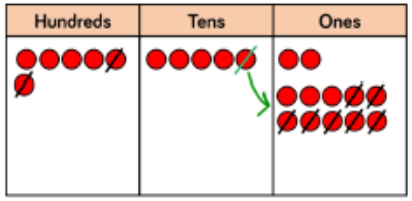
$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ 1 \end{array}$$



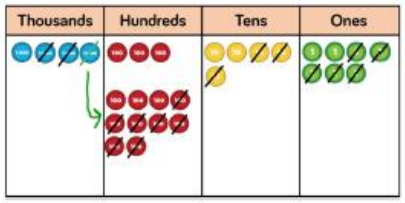
$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

Subtraction

Children should first subtract without regrouping before moving on to addition with regrouping. When building the model, children should just make the minuend using counters, then they subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and regroup e.g. change 1 ten for 10 ones. They can then subtract efficiently.



$$\begin{array}{r} {}^{4}\cancel{6}^{1}52 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\cancel{4}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

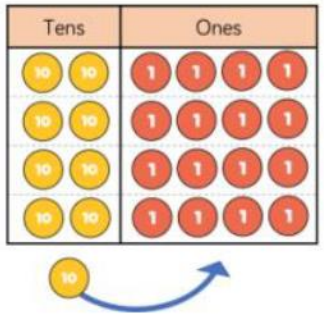
Multiplication

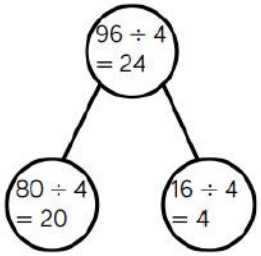
As numbers become larger in multiplication or the amounts of groups becomes higher, Deines becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$

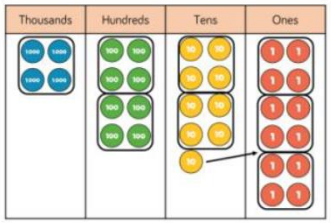
Division

Using place value counters is an effective way to support children's understanding of division. When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they regroup the counters e.g. change one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.





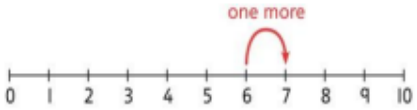
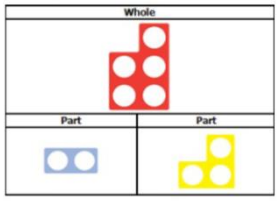
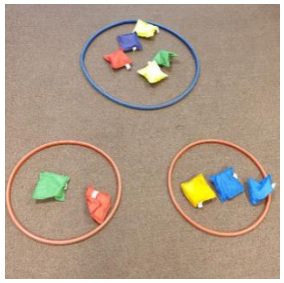
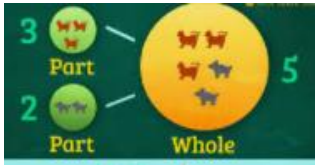
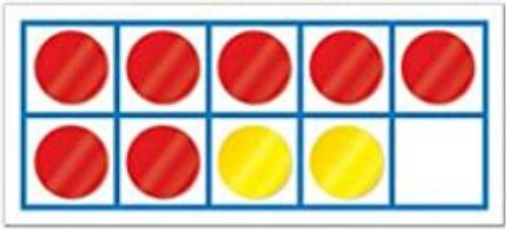
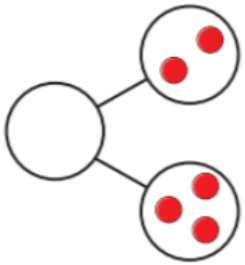
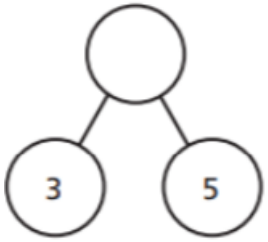
Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they regroup the counters e.g. change one hundred for ten tens.

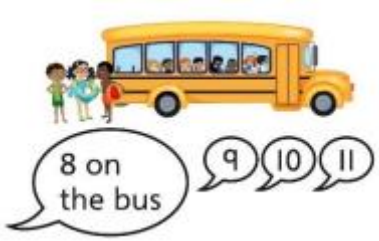
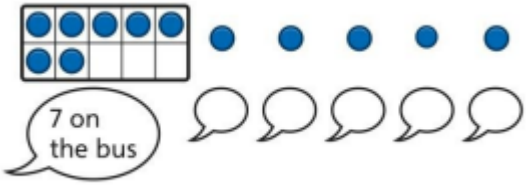
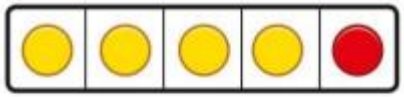
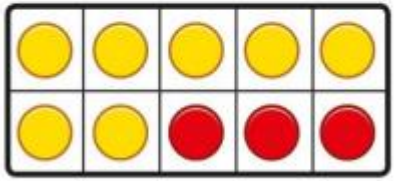
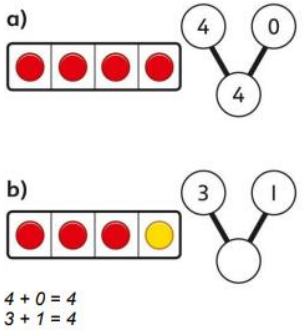


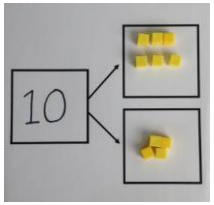
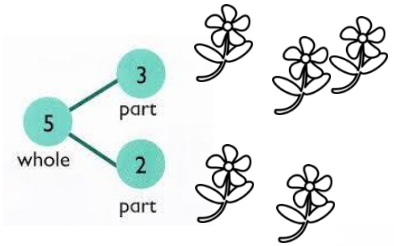
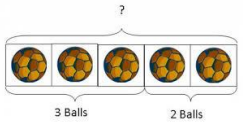
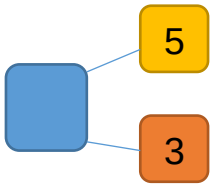
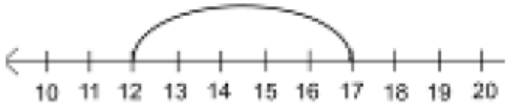
$$\begin{array}{r} 1223 \\ 4\overline{)\,4892^{1}} \end{array}$$

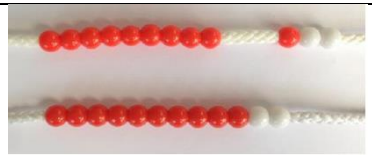
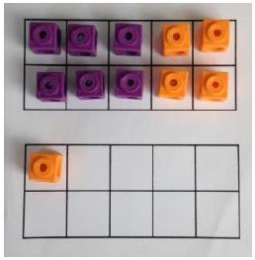
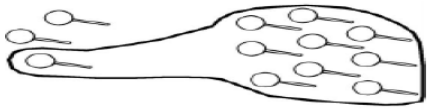
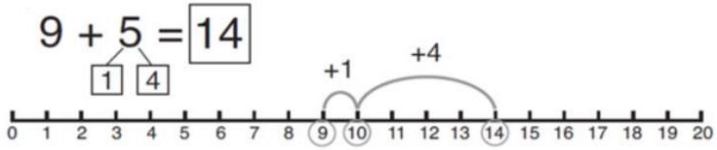
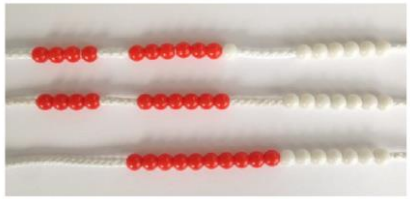
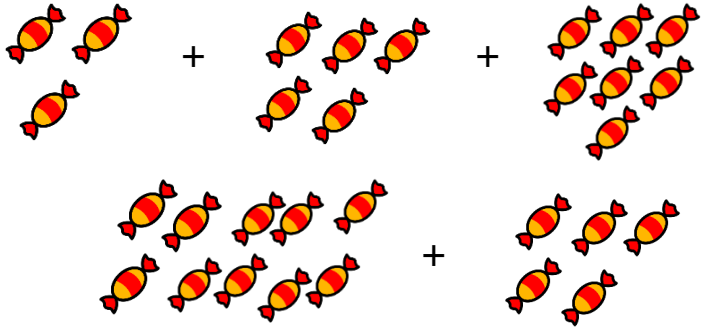
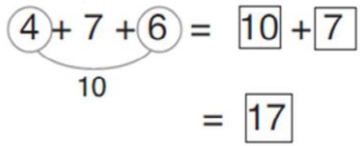
Progression in Calculations

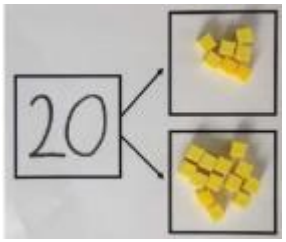
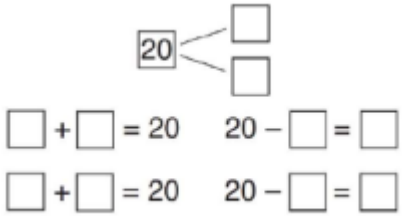
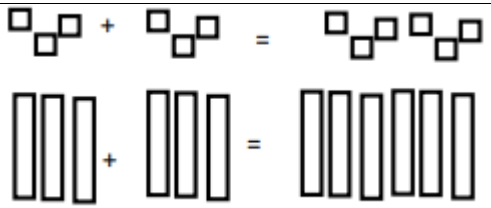
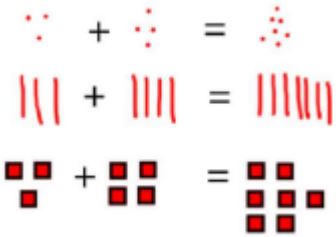
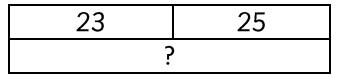
Addition

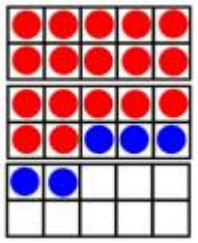
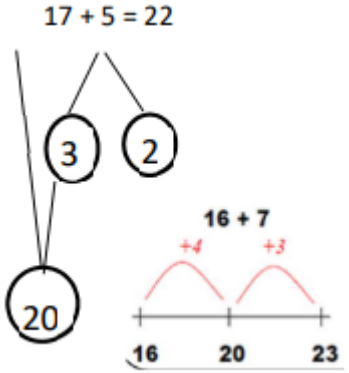
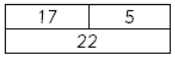
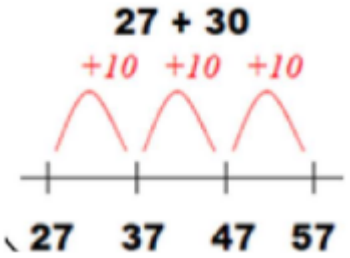
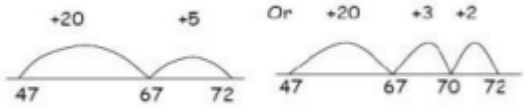
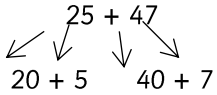
RECEPTION Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Counting and adding more	Add one more person or object to a group to find one more. Use a range of manipulatives e.g.  One more than 4 is 5	Add one more cube, counter or object to a group to represent one more. Draw own representations.  One more than 6 is 7	Use a number line to understand how to link counting on with finding one more.  One more than 6 is 7 7 is one more than 6
Explore part whole relationship – combining two parts to make a whole.	Use a range of manipulatives to explore combining two parts to make a whole.    	Draw own representations.  The parts are 2 and 3. The whole is 5. 	 3 + 5 = 8  6 + 4 = 10

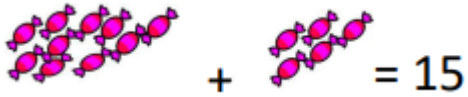
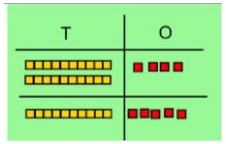
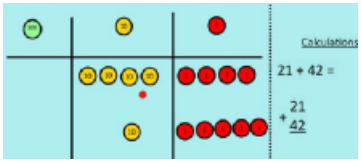
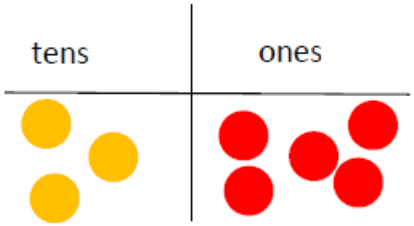
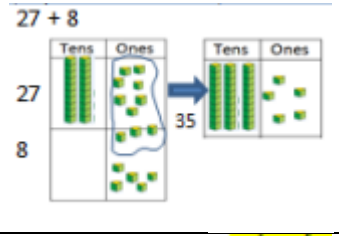
Adding by counting on.	Use knowledge of counting to 20 to find a total by counting on using people or objects. 	Use counters to support and represent their counting on strategy. 	
Knowing and finding number bonds within 10.	Break apart a group and put back together to find and form number bonds.  3 + 4 = 7  6 = 2 + 4	Use five and ten frames to represent key number bonds.  5 = 4 + 1  10 = 7 + 3	Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero. E.g. 
Solving problems using concrete, pictorial images	Sarah has 2 apples. Jon has 5 apples. How many apples do they have altogether? 		2 + 5 = 7

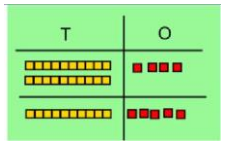
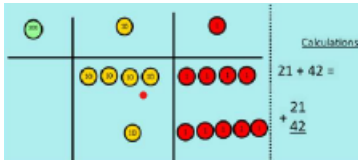
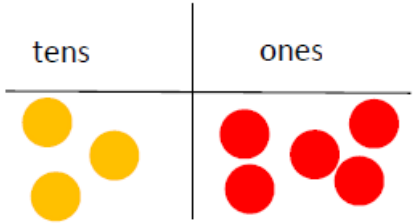
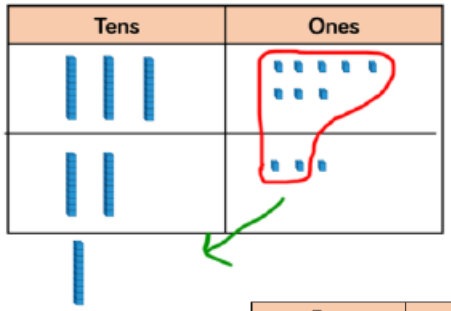
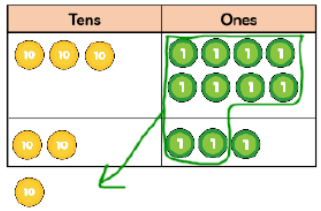
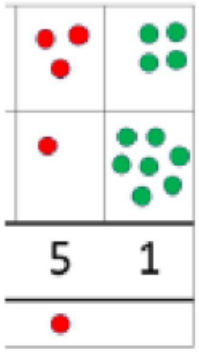
YEAR 1 Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	Use cubes to add two numbers together as a group or in a bar.    	Use pictures to add two numbers together as a group or in a bar.   	Use the part-part whole diagram as shown above to move into the abstract.  4 + 3 = 7 10= 6 + 4
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

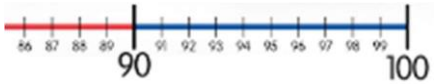
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 4 = 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	 Combine the two numbers that make 10 and then add on the remainder.

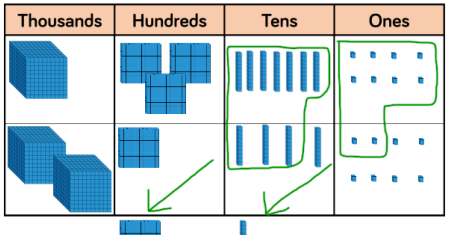
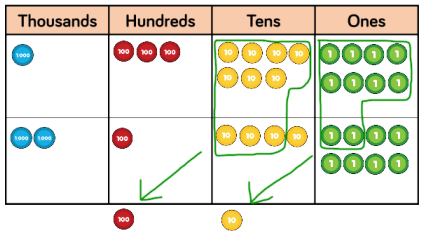
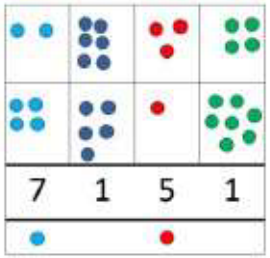
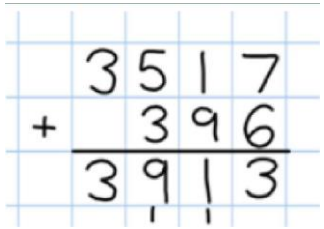
YEAR 2 Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Adding multiples of 10	50 + 30 = 10  Model using Dienes and bead strings	 3 tens + 5 tens = ____ tens 30 + 50 = ____ Use representations for base 10.	20 + 30 = 50 70 = 50 + 20 40 + ☐ = 60
Using know number facts (part, part, whole)	 Explore ways of making numbers within 20		Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtraction and use this to check calculations. ☐ + 1 = 16 16 – 1 = ☐ 1 + ☐ = 16 16 – ☐ = 1
Using known facts		 Children draw representations of H, T and O	3 + 4 = 7 leads to 30 + 40 = 70 Leads to 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

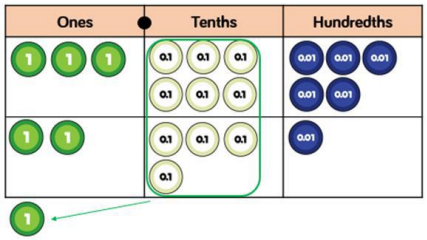
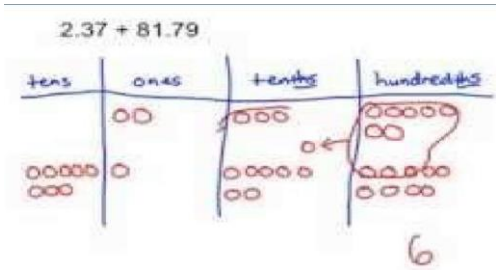
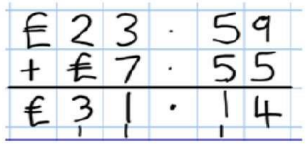
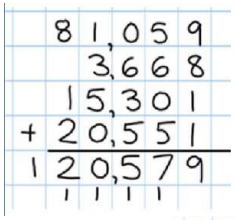
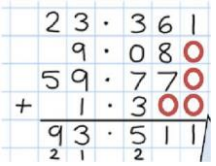
Add a 2 digit number and ones	 17 + 5 = 22 Children explore patterns e.g. 17 + 5 = 22 27 + 5 = 32	 Use part, part, whole and number line to model	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 – 5 = 17 22 – 17 = 5  Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a 2 digit numbers and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57
Adding two 2 digit numbers	 25 + 47 = 72 15 Model using dienes and place value counters	 Use a number line and bridge ten using part whole if necessary	 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72 Lead into recording column format, to reinforce place value and prepare children for formal written methods with larger values.

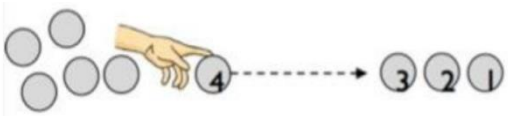
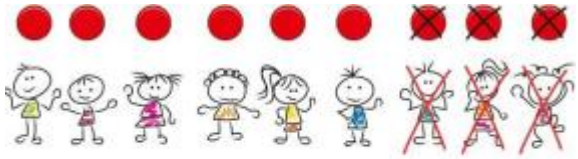
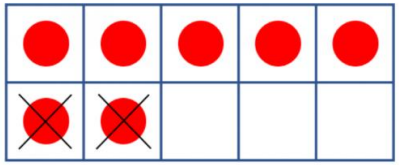
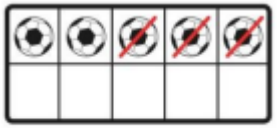
Add three 1 digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit	 Regroup and draw representation 	4 + 7 + 6 10 10 + 7 = 17 Combine the two numbers that make/bridge 10 then add on the third
Column Addition – no regrouping Add two digit numbers.	 Add together the ones first, then the tens.  Move to using place value counters.	Children move to drawing the counters using a tens and one frame. 	2 5 + 3 4 5 9 Add the ones first, then the tens.
Column Addition – regrouping Add up to 2 two digit numbers.	Exchange ten ones for a ten. Model using dienes and place value counters. 	Children draw own representations. 15 + 36 	Formal method in columns showing the regrouping underneath. 1 5 + 3 6 5 1 1

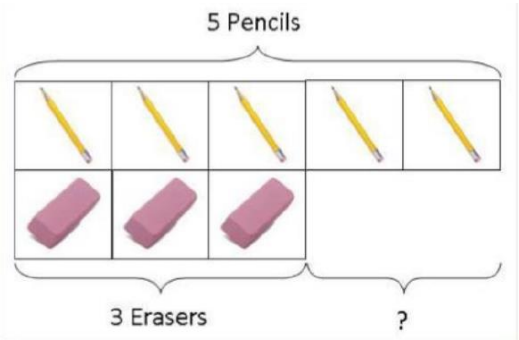
YEAR 3 Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Column Addition – no regrouping Add two or three 2 or 3 digit numbers.	 Add together the ones first, then the tens.  Move to using place value counters.	Children move to drawing the counters using a tens and one frame.  	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds
Column Addition with regrouping	Exchange ten ones for a ten. Model using dienes and place value counters.  38 + 23 = 61  	 3 4 +1 7 5 1 Children can draw a representation of the grid to further support their understanding, regrouping and putting the ten **underneath** the line.	20 + 8 40 + 5 60 + 13 = 73 Start by partitioning the numbers before formal columns to show the regrouping. 5 3 8 + 8 5 6 2 1 1 1

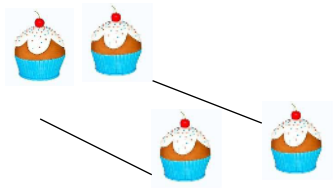
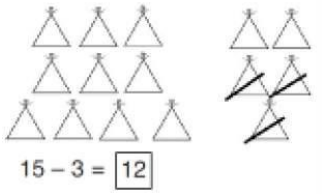
Estimate the answers to questions and use inverse operations to check answers	Estimating 98 + 17 = 100 + 20 - 120 	Use numbers lines to illustrate estimation. 	Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

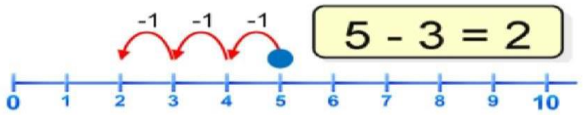
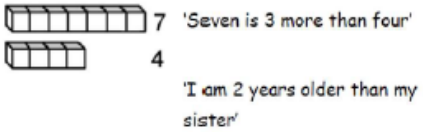
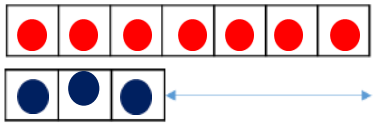
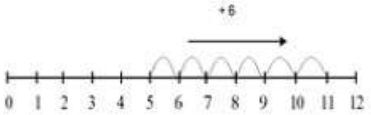
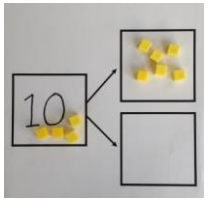
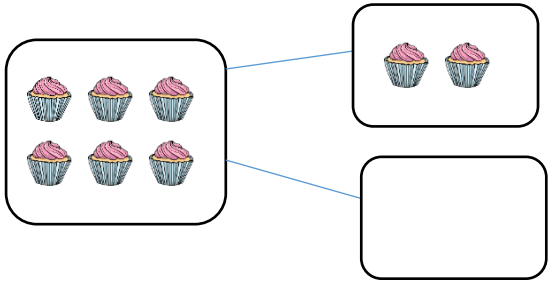
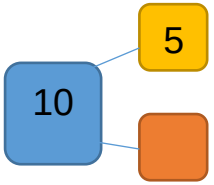
YEAR 4 - 6 Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Estimate and use inverse operations to check answers to a calculation	As per Year 3		
Y4 – add numbers with up to 4 digits	Children continue to use dienes or value counters to add, regrouping ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.  	Draw representations using place value grids. 	 Continue from previous work to regroup hundreds as well as tens. Relate to money and measures.

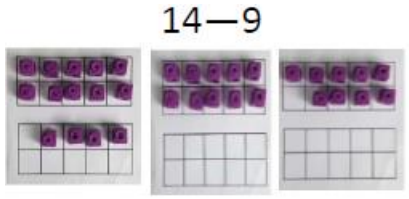
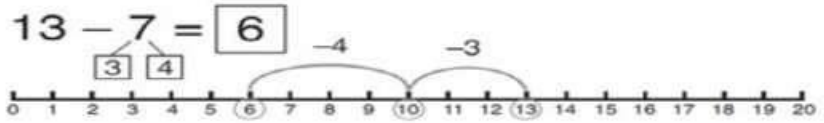
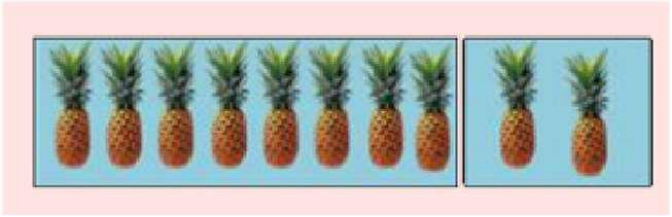
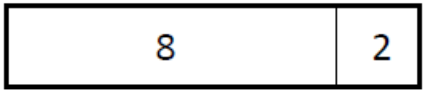
Y5 – add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model regrouping for addition.	 	72.8 +54.6 127.4 1 1 
Y6 – add several numbers of increasing complexity, including adding money, measures and decimals with different numbers of decimal points.	As Year 5	As Year 5	Insert zeros for place holders  

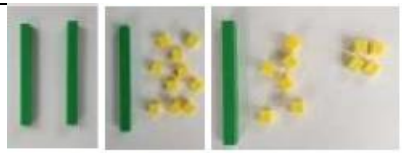
RECEPTION Subtraction			
Objective and Strategies	Concrete	Pictorial	Abstract
Counting back and taking away.	Taking away after counting out practical equipment. E.g. counters, multi-link cubes Children should be encouraged to physically remove objects using touch counting. By touch counting and dragging in this way, it allows children to keep track of how many they are removing. They will then touch count the amount that are left to find the answer.  4 less than 9 is 5 9 subtract 4 is 5	Children draw and cross out or use counters to represent objects from a problem.  9 – ☐ = ☐ There are ☐ children left.	Children count back to take away and use a number line or number track to support the method.  9 – 3 = 6
Subtraction within 10	Children should use enlarged ten frames and practical equipment.  8 – 4 = They should physically remove the ones they are taking away. Use a bead string to subtract 1s efficiently.  5 – 3 = 2	Draw own representations on tens frames and cross out the ones they are taking away. 7 – 5 = 2   5 – 3 = 2	Moving on to seeing the abstract calculation alongside the pictorial image.

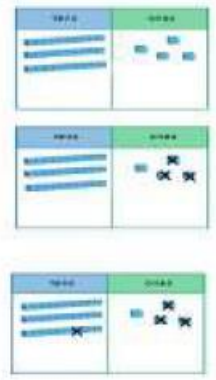
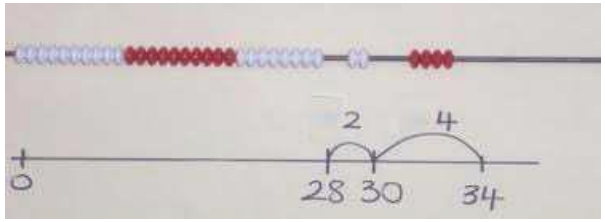
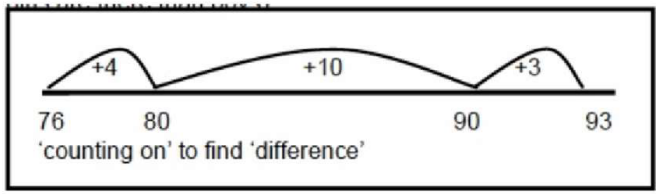
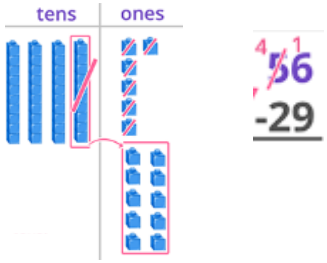
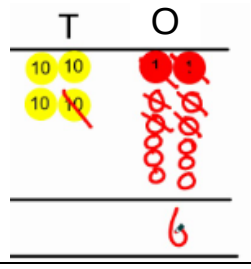
Solving problems using concrete and pictorial images	e.g. Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have? Children should use concrete objects to physically compare the objects.	Moving on to pictorial images. 	

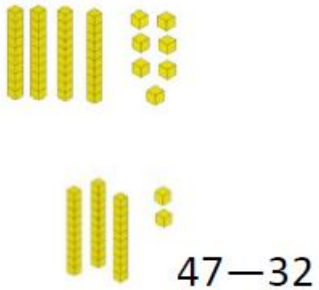
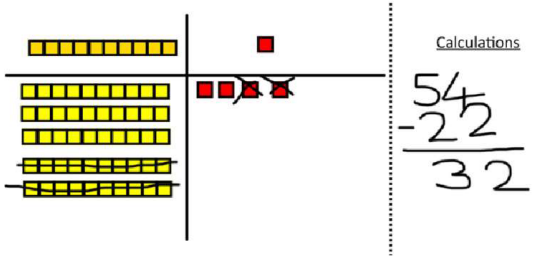
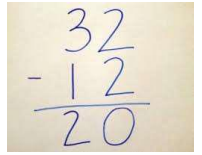
YEAR 1 Subtraction			
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  4 – 2 = 2	Cross out drawn objects to show what has been taken away. 	7 – 4 = 3 16 – 9 = 7

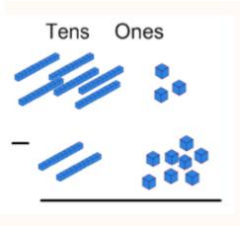
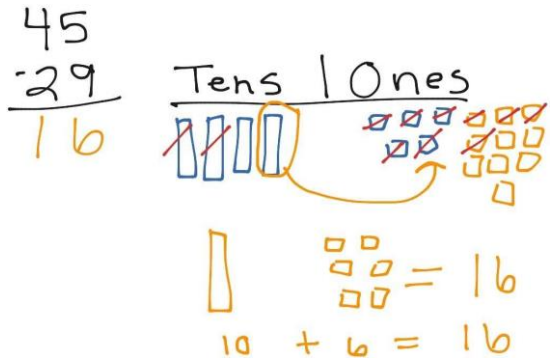
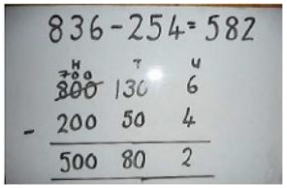
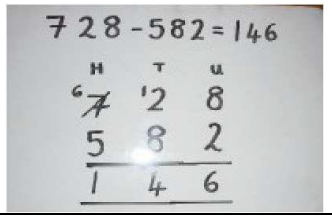
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Finding the difference	Compare objects and amounts.  Lay out objects to represent bar models.  There are 4 more red counters.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?
Represent and use number bonds and related subtraction facts within 20 Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

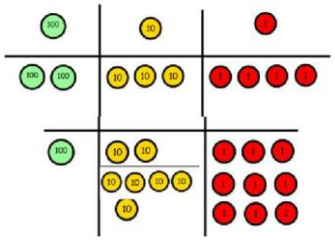
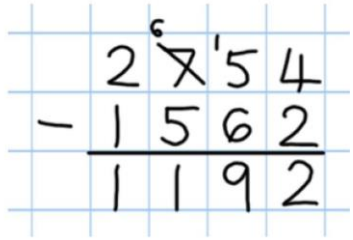
Make 10	 Make 14 on the ten frame. Take 4 away to make 10, then take 1 more away so that you have taken 5.	 13 – 7 Jump back 3 first, then another 4. Use 10 as the stopping point.	16 – 8 How many do we take off first to get to 10? How many left to take off?
Bar model Including the inverse operations	5 – 2 = 3 	10 – 2 = 8 	 10 = 8 + 2 10 = 2 + 8 10 – 2 = 8 10 – 8 = 2

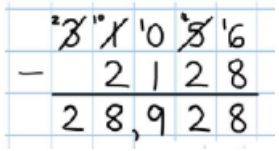
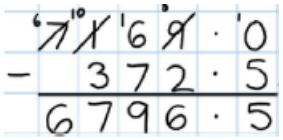
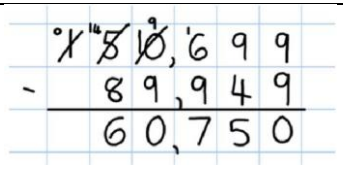
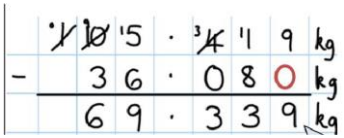
YEAR 2 Subtraction			
Objective and Strategies	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a place value chart to show how to regroup a ten into ten ones. Use the term 'take and make'.	 20 – 4 =	20 – 4 = 16

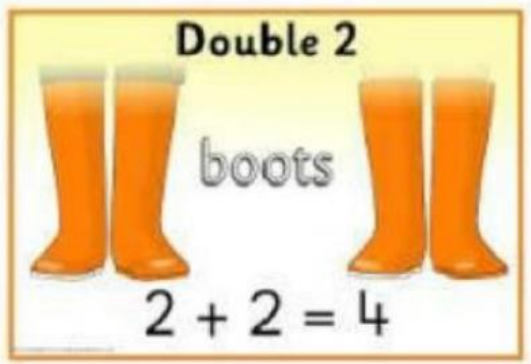
Partitioning to subtract without regrouping *'Friendly numbers'*	 34 – 13 = 21 Use dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of dienes and cross off.  43 – 21 = 22	43 – 21 = 22
Make ten strategy	 34 – 28 Use bead strings to model counting to the next ten and then the rest.	 93 – 76 Use a number line to count on to the next ten and then the rest.	93 – 76 = 17
Column subtraction with regrouping	Regroup, changing a ten for ten ones. Model using dienes and place value counters. 46 - 29 =  	Children draw own representations. 42 – 26 =  	Formal column method modelled alongside concrete and pictorial.

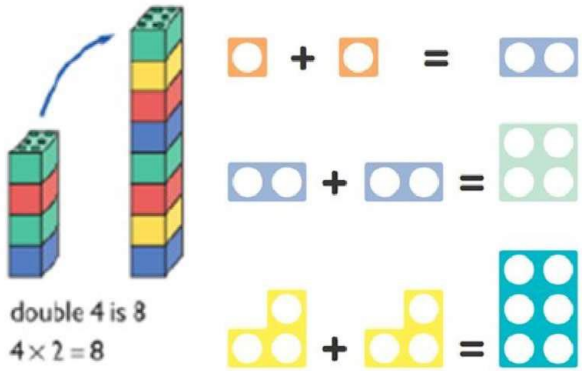
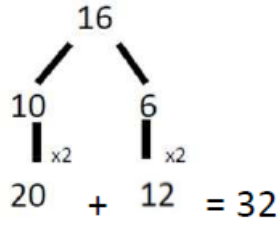
YEAR 3 Subtraction			
Objective and Strategies	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number - ones three digit number - tens three digit number - hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ___ - 1 688 – 10 = ___ 678 = ___ - 100
Column subtraction without regrouping *'Friendly numbers'*	 Use dienes to model.	 Draw representations to support understanding.	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 Intermediate step needed to lead to clear subtraction understanding. 

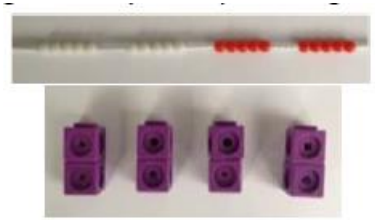
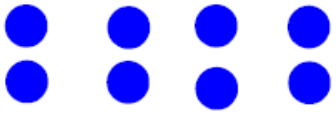
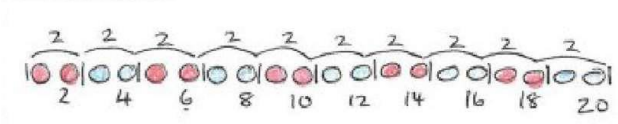
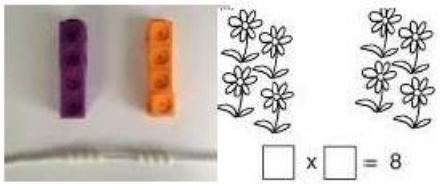
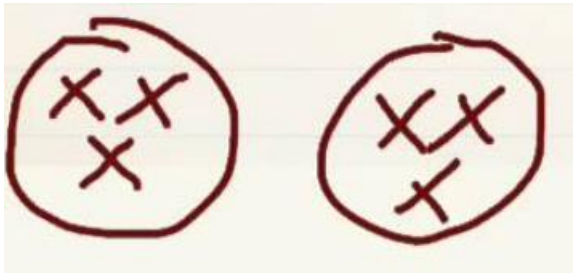
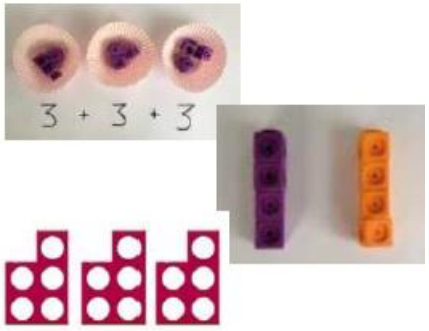
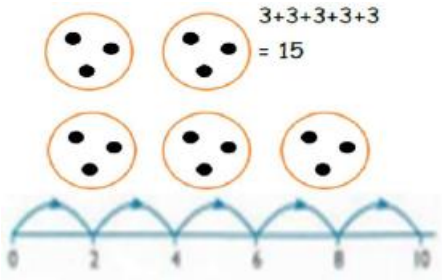
Column subtraction with regrouping	 Begin with dienes. Move to place value counters, modelling the regrouping of a ten into ten ones.	Children draw own representations of dienes or place value counters and cross off. 	Begin by partitioning into place value columns.  Then move to formal method. 

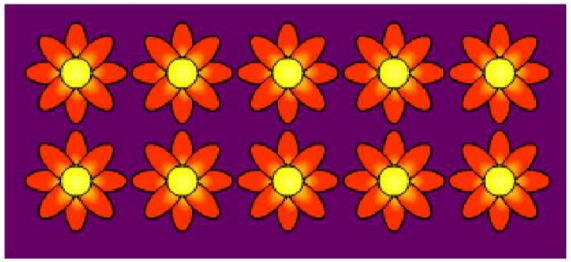
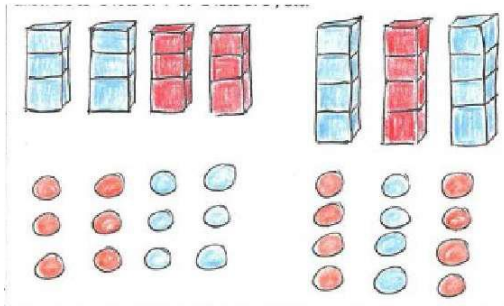
YEARS 4 - 6 Subtraction			
Objective and Strategies	Concrete	Pictorial	Abstract
Year 4 - Subtract with up to 4 digits *Introduce decimal subtraction through context of money*	234 – 179  Model process of regrouping using dienes and place value counters.	Children to draw place value counters and show their regrouping – See Year 3	

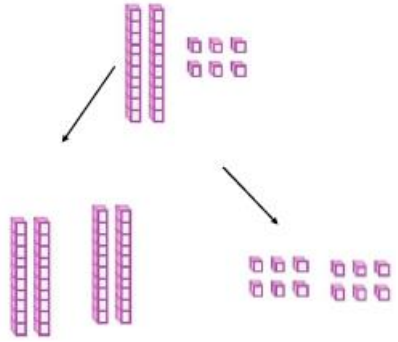
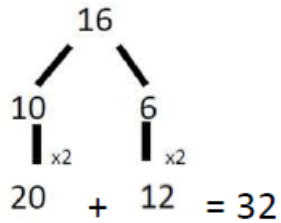
Year 5 – Subtract with at least 4 digits, including money and measures *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal – up to 3 decimal places*	As Year 4	Children to draw place value counters and show their regrouping– See Year 3	 Use zeros for placeholders. 
Year 6 – Subtract with increasingly large and more complex numbers and decimal values – up to 3 decimal places	As Year 4	Children to draw place value counters and show their regrouping – See Year 3	 

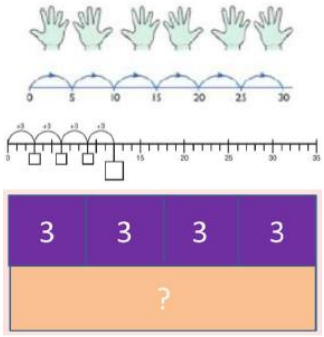
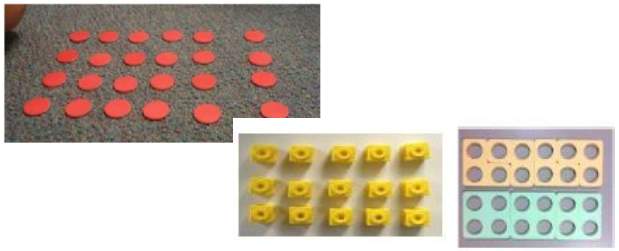
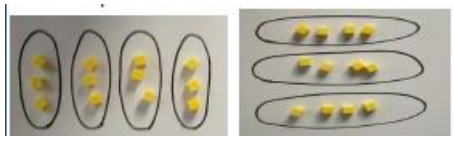
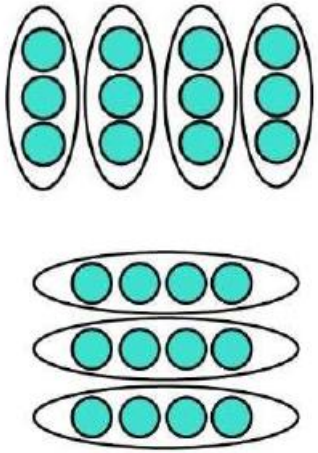
RECEPTION Multiplication			
Objective and Strategies	Concrete	Pictorial	Abstract
Experiencing equal groups of objects Doubling when solving practical problems	Children will experience groups of objects.  There are 6 pairs of socks. How many socks are there altogether?	Understand doubling as repeated addition. 	

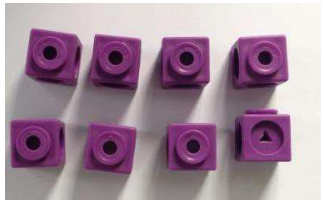
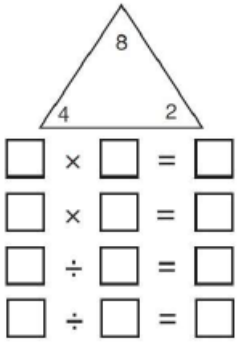
YEAR 1 Multiplication Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication			
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. Double 4 is 8 	Partition a number and then double each part before recombining it back together. 

Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting. Children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	Use manipulatives to create equal groups. 	Draw and make representations. 2 groups of 3 	2 x 4 = 8
Repeated addition	Use different objects to add equal groups. 	Use pictorial, including number lines, to solve problems. There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 

Understanding arrays	Use objects laid out in arrays to find the answer to 2 lots of 5, 3 lots of 2 etc 	Draw representations of arrays to show understanding. 	3 x 2 = 6 2 x 5 = 10

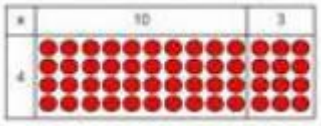
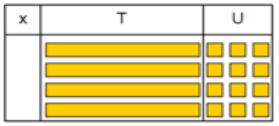
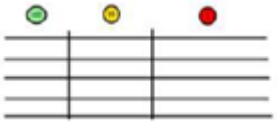
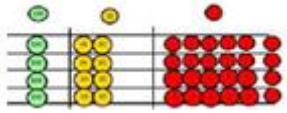
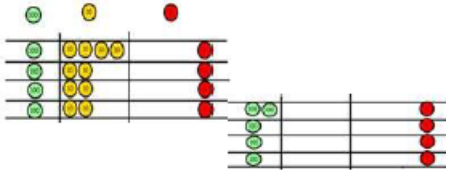
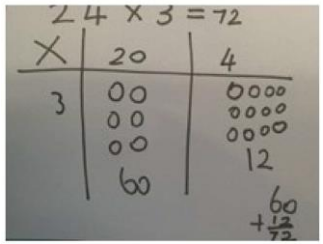
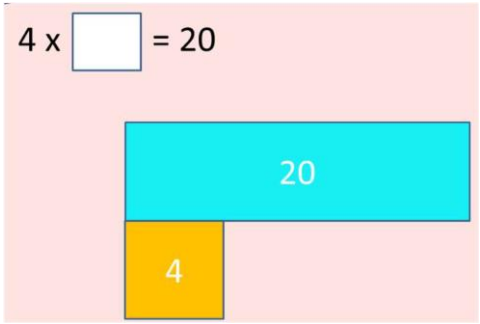
YEAR 2 Multiplication Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.			
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and place value counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part before recombining it back together. 

Counting in multiples of 2, 3, 4, 5, 10 from 0 (Repeated addition)	Count the groups as children are skip counting. Children may use their fingers as they are skip counting. Use bar models.  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30
Multiplication is commutative	Create arrays using counters, cubes and numicon.  Children should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity. 	12 = 3 x 4, 12 = 4 x 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Using the inverse *This should be taught alongside division so pupils learn how they work alongside each other.*	 2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2		Show all 8 related fact family sentences. 2 x 4 = 8 8 ÷ 2 = 4 8 = 2 x 4 4 = 8 ÷ 2 4 x 2 = 8 8 ÷ 4 = 2 8 = 4 x 2 2 = 8 ÷ 4

YEAR 3 Multiplication

Children should be able to recall and use multiplication and division facts for the 3, 4 and 8 times tables.

Objective and Strategies	Concrete	Pictorial	Abstract
Grid method progressing to the formal method Multiply 2 digit numbers by 1 digit numbers	Show the links with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to dienes to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows. 4 x 126  Fill each row with 126.  Add up each column, starting with the ones completing any regrouping needed. Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar models can be used to explore missing numbers.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
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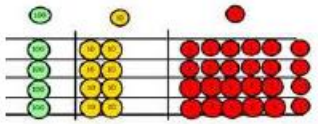
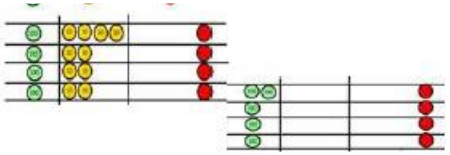
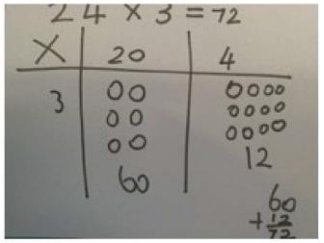
7	210	35	 **210 + 35 = 245** Move forward to the formal written method. 3 5 X 7 2 4 5 3

Solve problems including missing number problems and integer scaling problems.			*Three times as high, eight times as long.* __ x 5 = 20 20 ÷ __ = 5

YEARS 4 - 6 Multiplication Children should be able to recall and use multiplication and division facts for all times tables by the end of year 4.			
Objective and Strategies	**Concrete**	**Pictorial**	**Abstract**
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (Year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126. Add up each column, starting with the ones completing any regrouping needed. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
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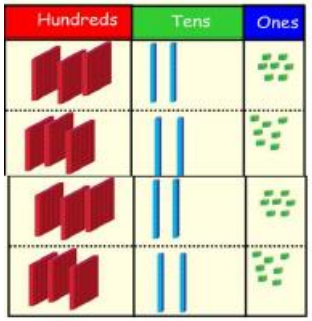
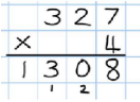
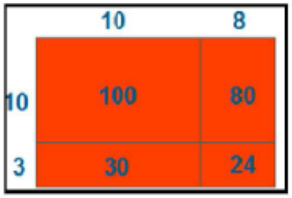
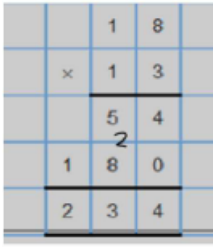
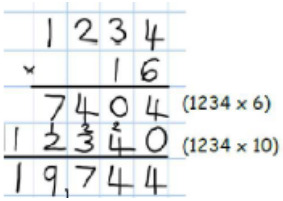
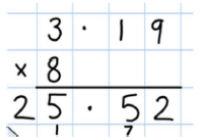
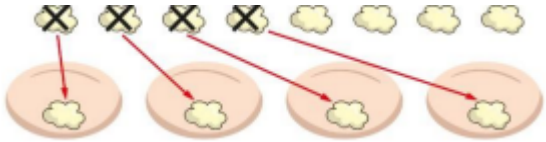
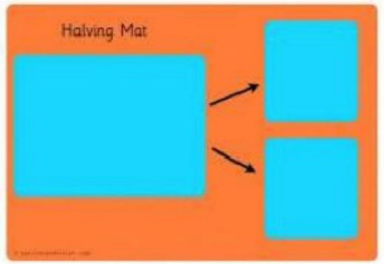
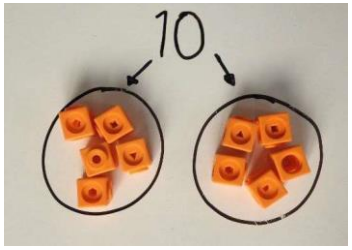
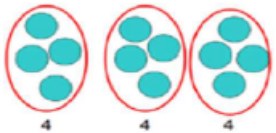
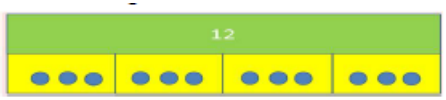
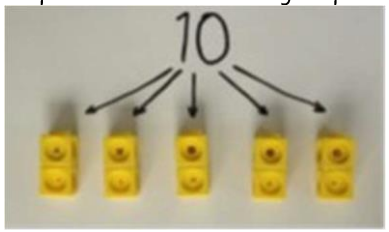
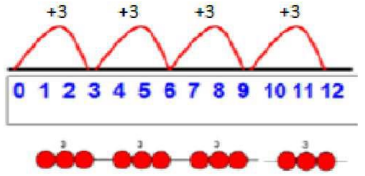
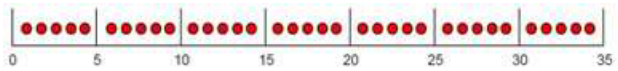
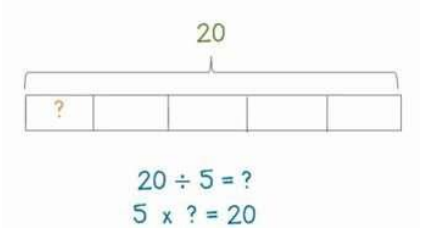
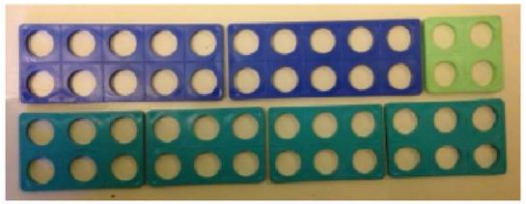
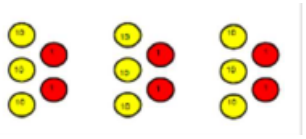
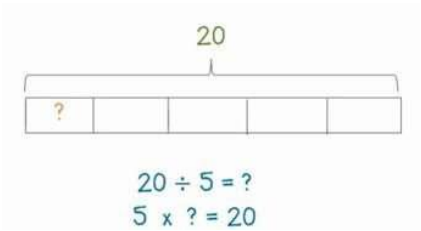
7	210	35	 **210 + 35 = 245**

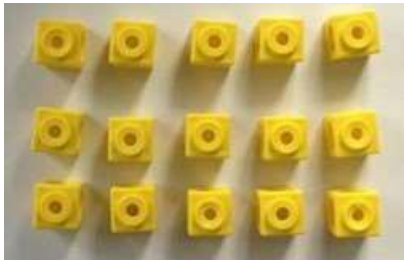
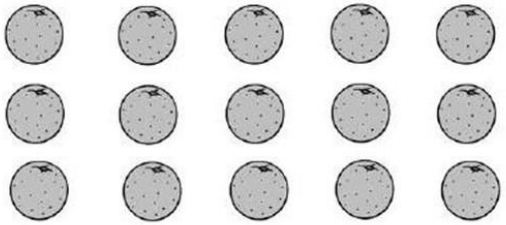
Column multiplication for 3 and 4 digits x 1 digit	 Children can continue to be supported by place value counters or dienes at this stage of multiplication. This should be done initially with no regrouping e.g. 321 x 2 = 642 It is important at this stage that children always multiply the ones first. The corresponding multiplication should be modelled alongside.	The grid method may be used to show how this relates to a more formal written method. | x | 300 | 20 | 7 | | --- | --- | --- | --- | | 4 | 1200 | 80 | 28 | Bar modelling can support children when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 This leads to a more compact method. 
Long multiplication for multiplying 2, 3 or 4 digit numbers by a 2 digit number	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	The grid method may be used to show how this relates to a more formal written method.  Continue to use bar modelling to support problem solving.	18 x 3 on the first row. 18 x 10 on the second row. Show multiplying by 10 by putting 0 in the ones column first.  
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer. 

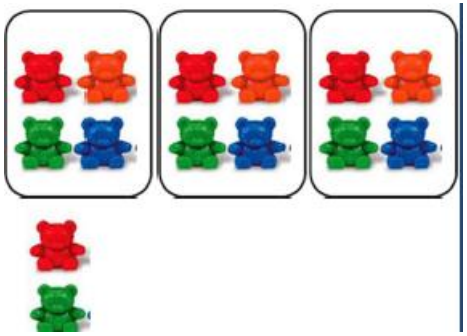
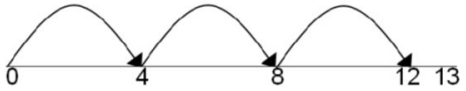
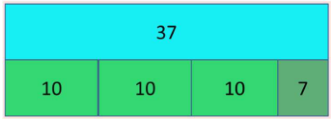
RECEPTION Division			
Objective and Strategies	Concrete	Pictorial	Abstract
Learning to make equal groups	Sort a whole set people and objects into equal groups.  There are 10 children altogether. There are 2 in each group. There are 5 groups.	Represent a whole and work out how many equal groups.  There are 10 in total. There are 5 in each group. There are 2 groups.	
Sharing objects practically. Hearing and being exposed to language to describe half and seeing visual representations.	Share a set of objects into equal parts and work out how many are in each part.  	  	

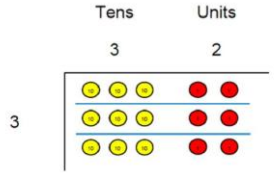
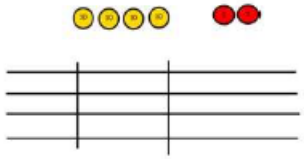
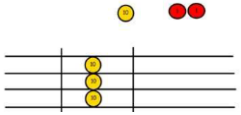
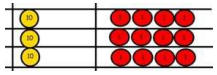
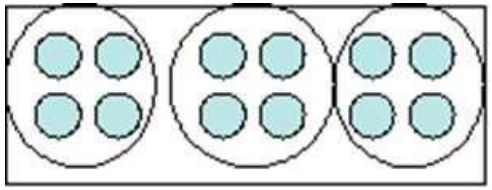
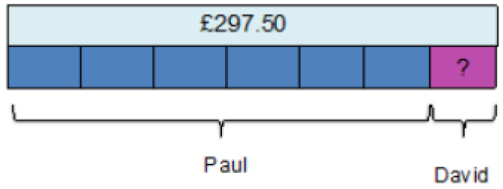
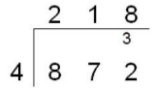
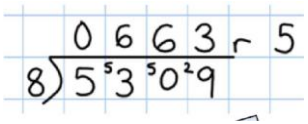
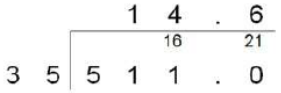
YEAR 1 Division			
Objective and Strategies	Concrete	Pictorial	Abstract
Division as sharing	  I have 10 cubes. Can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 shared between 2 is 4  12 shared between 3 is 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 shared between 3 is 4. 12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. Split 10 cubes into 5 groups. 	Use number lines for grouping.  **12 ÷ 3 = 4**	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

		Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be in each group. 	

YEAR 2 Division			
Objective and Strategies	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to support solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4

Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

YEAR 3 Division			
Objective and Strategies	Concrete	Pictorial	Abstract
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a reminder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 r5

YEARS 4 - 6 Division			
Objective and Strategies	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit Short division	Use place value counters to divide using the bus stop method alongside. 96 ÷ 3 =  42 ÷ 3 =  Start with the biggest place value. We are sharing 40 into 3 groups. We can put 1 ten into each group and we have 1 ten left over.  We regroup, changing this ten for ten ones and then share the ones equally among the groups.  We look at how much is in 1 group. The answer is 14.	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. Continue to use the bar model to support division. E.g. Paul and David buy a car together at a cost of £297.50. Paul pays 6 times more than David. How much does David pay? 	Begin with divisions that divide equally with no remainder.  Move on to divisions with a remainder.  Finally move into decimal places to divide the total accurately. 

Divide 4 digit numbers by a 2 digit number (Year 6) Long division